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The Application of Random  
Search Theory to the Detection  
of Tactical Ballistic Missile Launchers

by

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## ABSTRACT

The search for Tactical Ballistic Missile (TBM) launchers is modeled mathematically using the Random Search Model. Special provisions are made in the model to account for the fact that the launcher is not always exposed to detection by the searcher. The probability that the launcher is exposed to detection is assumed to be both deterministic and stochastic. The tactical implications of the results of the model are discussed and several methods are provided to improve search performance based on an analysis of the model parameters.

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## TABLE OF CONTENTS

I.	INTRODUCTION . . . . .	1
A.	BACKGROUND . . . . .	1
B.	MOBILE TBM LAUNCHER CIRCULATION MODEL . . . . .	2
C.	LESSONS LEARNED FROM THE PERSIAN GULF WAR . . . . .	4
D.	STATEMENT OF THESIS . . . . .	6
E.	THESIS OUTLINE . . . . .	6
II.	PROBLEM SCOPE . . . . .	7
A.	DEFINING THE PROBLEM . . . . .	7
B.	THE TARGET . . . . .	8
C.	SEARCHER CHARACTERISTICS . . . . .	10
D.	SIMILARITIES TO ANTI-SUBMARINE WARFARE . . . . .	11
E.	DIFFERENCES BETWEEN TBM LAUNCHERS AND SUBMARINES . . . . .	13
III.	METHODOLOGY . . . . .	16
A.	BASIC APPROACH . . . . .	16
B.	RANDOM SEARCH MODEL . . . . .	17
C.	MODIFIED RANDOM SEARCH FORMULATION . . . . .	20
D.	EXTENSION TO SIMULTANEOUS MULTIPLE SEARCHERS . . . . .	22

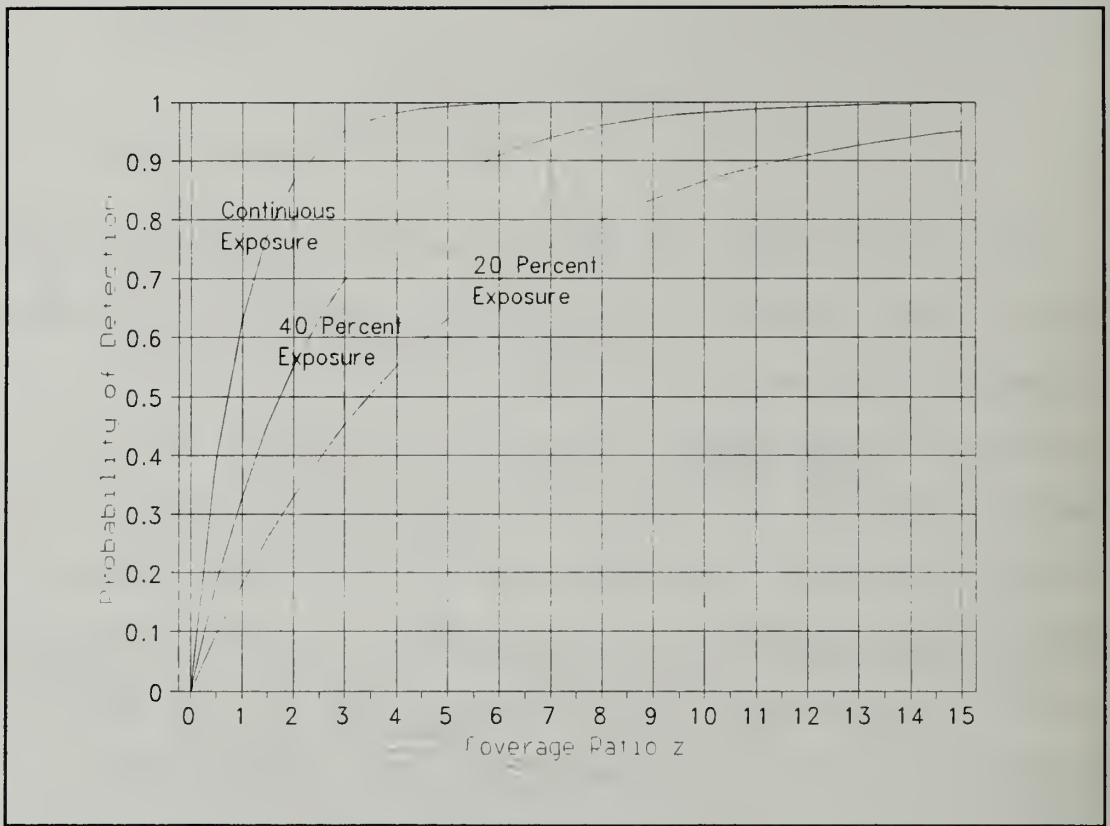
IV.	ANALYSIS OF MODIFIED RANDOM SEARCH MODEL . . . . .	24
A.	DETERMINISTIC EXPOSURE . . . . .	24
B.	UNIFORMLY DISTRIBUTED EXPOSURE . . . . .	27
C.	TRIANGULARLY DISTRIBUTED TARGET EXPOSURES . . . . .	28
	1. Triangular Distribution with Mean of One-half . . . . .	28
	2. Triangular Distribution with Mean of One-Third . . . . .	31
D.	SUMMARY . . . . .	33
V.	TACTICAL IMPLICATIONS OF SEARCH MODEL . . . . .	36
A.	FACTORS AFFECTING THE COVERAGE RATIO . . . . .	37
	1. Sweep Width . . . . .	37
	2. Total Search Length . . . . .	38
	3. Search Area . . . . .	39
B.	FACTORS AFFECTING TARGET EXPOSURE TO DETECTION . . . . .	40
	1. Use of Passive Sensors . . . . .	40
	2. Use of Active Sensors . . . . .	41
	3. Use of Area Denial Weapons . . . . .	42
	4. Destruction of Roads and Bridges . . . . .	42
C.	MULTIPLE SEARCHERS . . . . .	43
D.	FACTORS OUTSIDE OF THE SEARCH MODEL . . . . .	43
	1. Historical Analysis of Operations . . . . .	44
	2. Cuing the Search Effort . . . . .	44
	3. Training and Practice . . . . .	45
	4. Destroy or Trail . . . . .	46

E.	SUMMARY . . . . .	47
VI.	SUMMARY AND CONCLUSIONS . . . . .	48
A.	SUMMARY . . . . .	48
B.	CONCLUSIONS . . . . .	49
C.	POSSIBLE FOLLOW-ON RESEARCH . . . . .	51
APPENDIX A.	DEVELOPMENT OF RANDOM SEARCH MODEL . . .	53
A.	RANDOM SEARCH MODEL . . . . .	53
APPENDIX B.	PROBABILITY OF DETECTION BY CONDITIONING	56
A.	$\Phi$ DISTRIBUTED UNIFORMLY BETWEEN ZERO AND ONE .	56
B.	$\Phi$ DISTRIBUTED TRIANGULARLY WITH MEAN ONE-HALF .	57
C.	$\Phi$ DISTRIBUTED TRIANGULARLY WITH MEAN ONE-THIRD	58
LIST OF REFERENCES	. . . . .	60
INITIAL DISTRIBUTION LIST	. . . . .	61

## EXECUTIVE SUMMARY

The proliferation of Tactical Ballistic Missiles (TBM's) poses a great threat to the national interests of the United States. It has been shown that even a modest improvement in attack operations against the ballistic missile launchers and support equipment can yield large dividends in terms of fewer missiles launched. To plan and analyze attack operations against TBM launchers, it is helpful to have mathematical models that describe the various phases of the operation. This thesis presents a mathematical model that is based on the theory of random search and describes the search for TBM launchers and support equipment in terms of the probability of detection and the required search effort. The model includes a specific term that accounts for the fact that the TBM launchers are not always in the open able to be detected by the searcher. This term, the probability of target exposure to detection, was considered to be both deterministic and random during the model analysis.

The probability of detecting a TBM launcher during a search depends on the probability that the target is exposed to detection and the coverage ratio, a measure of search effort. The figure below shows this relationship for a few values of exposure probability. Notice that to achieve a 90



Deterministic Exposure Proportions.

percent probability of detecting a continuously exposed launcher requires a coverage ratio of about 2.5. This means the search area has to be covered two and a half times during the search. To achieve the same probability of detection against a launcher that is only exposed 20 percent of the time a coverage ratio of over 11 is required, almost a five fold increase in required search effort. If the searcher knew when the launcher was exposed to detection, he could reduce the search effort by five times by only searching during those periods.

Actions by the searcher that either increase the launcher's exposure probability or decrease the time searched

when the launcher is hidden, raise the probability of detecting the launcher or reduce the required search effort.

Such actions could include:

- developing better sensors, preferably passive, which allow for the detection of TBM launchers at greater ranges
- increasing the time on station capabilities of search assets
- reducing the area to be searched by using computer aided search models to identify likely operating areas
- using area denial weapons like mines to increase the amount of time the launchers are exposed to detection
- destroying roads and overpasses in TBM launcher operating areas to increase the exposure time
- developing and using remote ground based sensors or human observers to better refine when and where to search
- increasing the number of searchers, which increases the level of effort
- using analysis of historical operations to refine when and where to search
- marking or trailing launchers once they are found if the expected benefit of finding a staging area is greater than the benefit of killing that launcher
- developing search tactics that make the search effort less random and more systematic

These suggestions can greatly improve the performance of friendly forces in reducing the threat from Tactical Ballistic Missiles.







## I. INTRODUCTION

### A. BACKGROUND

Defense against Tactical Ballistic Missile (TBM) attack is of great interest to the Department of Defense and the National Command Authority. Secretary of Defense Les Aspin as the Chairman of the House Committee on Armed Services addressed Tactical Missile Defenses (TMD) in the following way:

The global proliferation of ballistic missile technology and weapons of mass destruction has become one of the most immediate and dangerous threats to U.S. national security in the post Cold War era. Over time, this threat will most likely evolve from today's shorter-range, inaccurate missiles in the direction of more sophisticated, longer-range and increasingly accurate systems. Therefore, the question of how the U.S. can modernize its TMD capabilities to best ensure that its forward deployed and power projection forces possess effective defenses against future tactical ballistic missile threats is paramount. [Ref. 1]

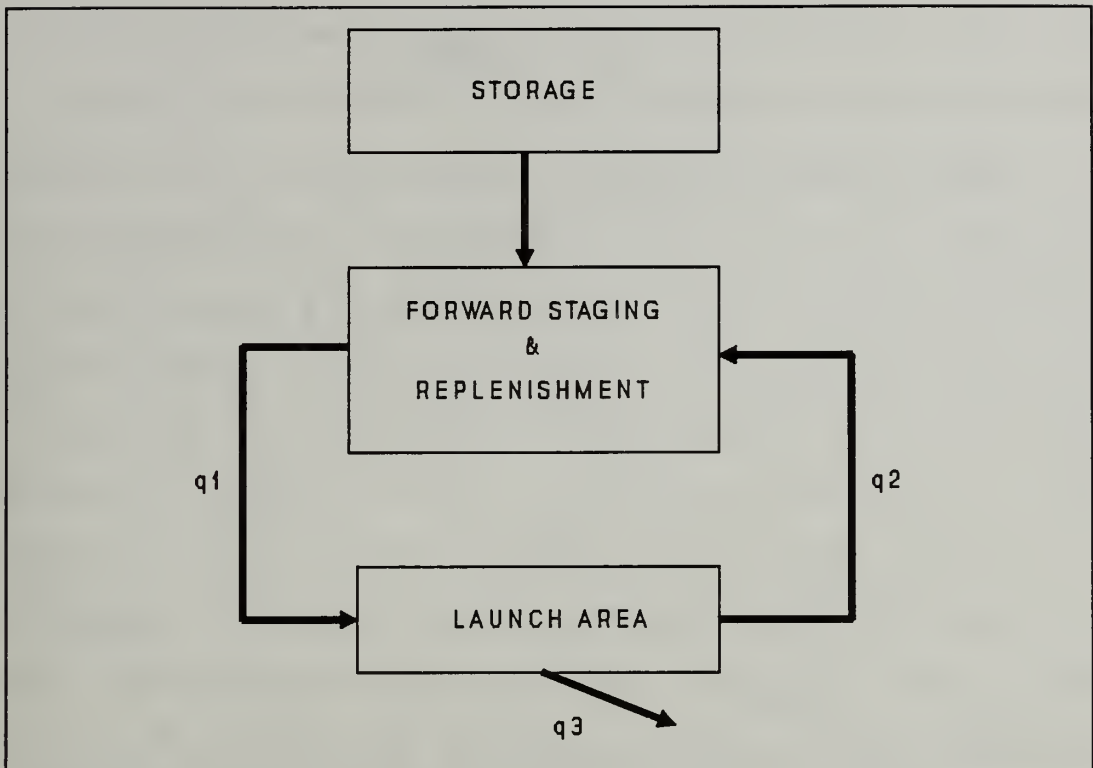
Tactical ballistic missile (TBM) defense can take many forms. The most visible form of defense is the active defense using systems like the PATRIOT and AEGIS to shoot down the ballistic missiles once in flight. Another type of defense is called attack operations or counter-force and it includes attacking the ballistic missile launchers, support equipment, storage sites and infrastructure. Finally, passive defenses like counter-targeting and civil defense can be used to mitigate the danger of an attack. The United States needs to examine

the various ways in which to defend its forces against Tactical Ballistic Missile attack.

Effectively countering the Tactical Ballistic Missile threat will require community identification with all aspects of the mission and detailed analytical thought. The campaign to counter the TBM threat should be very similar to the allied campaign against the U-boats of World War II. The campaign against the U-boats was characterized by dedicated organizations that were responsible for all aspects of the campaign. This included procurement of assets, the training of operators, and detailed analysis of operations to improve performance. The formal study of Operations Research developed from these analytical efforts and the science of Search Theory was started by the Anti-Submarine Warfare Operations Research Group (ASWORG) of the United States Navy. This close tie between a dedicated warfare community and Operations Analysis has been the defining characteristic of Anti-Submarine Warfare in the U.S. Navy. A similar union is needed to counter the TBM threat.

#### **B. MOBILE TBM LAUNCHER CIRCULATION MODEL**

Lt. Ehlers developed a circulation model to analyze the operations of mobile TBM launchers [Ref. 2]. The model was based on a 1969 Center of Naval Analyses study of a steady state anti-shipping campaign conducted by independently operating submarines. Figure 1 shows the circulation model



**Figure 1.** Mobile TBM Launcher Circulation Model.

developed by Lt. Ehlers. He used the circulation model to show the benefit of a counter-force effort. In this circulation model  $q_1$  is the outward transit survival probability and  $q_2$  is the return transit survival probability.  $q_3$  is the probability that a missile is not destroyed in flight, the missile survival probability. Using this model it was shown that the expected number of enemy missiles  $T$  that "successfully penetrate the air defense network and reach their targeted area from all  $n$  enemy launchers" [Ref. 2:p. 12] is

$$E[T] = \frac{nq_1q_3}{1 - q_1q_2}$$

The analysis of the model showed that if  $q_2 = 0.9$  and the outward transit survival probability  $q_1$  is reduced just 10 percent (from 1.0 to 0.9) then the expected number of missiles reaching the target area will be reduced by more than 50 percent for all values of  $q_3$ . This highlights the benefit of hitting the TBM launchers vice just the missiles in flight.

Hitting the launchers becomes even more important if the ballistic missiles are armed with multiple chemical or biological warheads. With multiple warheads the active defenses could become saturated or the warheads may be too small to hit. Additionally, intercepting a ballistic missile armed with a chemical or biological warhead may just release the contents a few hundredths of a second early and would not protect the target area. The proliferation of ballistic missiles and warheads of mass destruction requires that the missiles and launchers be attacked on the ground before launch to ensure the safety of friendly forces and civilians.

### C. LESSONS LEARNED FROM THE PERSIAN GULF WAR

During the Persian Gulf War the air campaign planners "intended to reduce the offensive [TBM] threat by attacking the fixed [TBM launch] sites, support bases, and production facilities, potential hide sites and support facilities for

mobile launchers, but not the launchers themselves." [Ref. 3:p. 19] The mobile launchers were not attacked because no one had "devised, prior to the war, a search-and-destroy scheme for dealing with the mobile launchers." [Ref. 3:p. 19] Even with the attacks on known targets Iraq was still able to average 14.7 launches per week during the war. Additionally, no mobile launchers were ever confirmed as destroyed by fixed wing aircraft [Ref. 3:p. 26]. The aircraft probably attacked decoys or vehicles such as tanker trucks that look like TBM launchers. The amount of effort applied to the destruction of mobile TBM launchers was significant but it was only a small part of the overall air campaign. There were roughly 1000 "Scud patrol" sorties that dropped ordnance on other targets and only about 215 attacks on mobile launchers themselves out of 1,500 (about 15 percent) total strikes associated with ballistic missile capabilities [Ref. 3:p. 27]. These air strikes represent less than 6 percent of the 41,310 total fixed-wing air strikes conducted during the war.

The effort to destroy mobile TBM launchers during the Persian Gulf War was largely ineffective. The United States needs to do better if it hopes to counter TBM attacks in future conflicts. The Navy started the study of Operations Research in World War II by opening the ASWORG group to find better ways to counter the German U-boat campaign [Ref. 4]. The science of Search Theory came out of that effort headed by Bernard Koopman [Ref. 5]. Fifty years later, the application



of Search Theory to the problem of finding mobile TBM launchers may provide valuable insights into the problem and help lead to possible solutions.

#### **D. STATEMENT OF THESIS**

In this thesis, the science of Search Theory, particularly the random search model, is applied to the problem of finding mobile Tactical Ballistic Missile launchers.

#### **E. THESIS OUTLINE**

Chapter II of this thesis further defines the problem of finding TBM launchers, describes the characteristics of the launchers and the searchers, and explores the similarities and differences between finding TBM launchers and U-boats for which Search Theory was developed.

In Chapter III the methodology of the analysis is discussed. This includes an introduction to the Random Search model and a description of how it must be modified to be relevant to searching for TBM launchers.

Chapter IV discusses the results of the modified search model.

Chapter V examines some of the tactical implications associated with the results of the modified search model.

Finally, in Chapter VI the thesis is summarized and ideas for follow-on research are presented.

## II. PROBLEM SCOPE

### A. DEFINING THE PROBLEM

As was seen in Chapter I, numerous approaches can be taken to the problem of defeating the tactical ballistic missile (TBM) threat. Additionally, it was seen by looking at the Circulation Model for TBM launchers, that even a modest improvement in the pre-launch search and destruction success will greatly reduce the maximum expected number of missiles launched and the number of missiles reaching the target area. The effort to destroy the launchers and associated support equipment is called attack operations or counter-force. While some work has been done in the counter-force area, the major emphasis has been in finding ways to shoot down the missiles. The counter-force effort should include search, detection, and destruction of TBM launchers, support equipment, and missile launch infrastructure. Recent work at the Naval Postgraduate School has concentrated on the search, detection, and destruction of TBM launchers before missile launch. Lt Hair in his March 1993 thesis [Ref. 6] looked at a search effort distribution problem. Specifically, he divided a large search area into smaller sub-areas and assigned to each sub-area a probability based on the likelihood that a TBM missile launcher was operating there. These probabilities were

assigned based primarily on the surface contours in the sub-area and the proximity to roads. Lt Hair then developed an optimum distribution of search effort based on these probabilities.<sup>1</sup> A similar effort to identify likely operating areas for TBM launchers was used during the Persian Gulf War with good results. This thesis looks at the overall search process through a mathematical model in an effort to identify strategies that will improve search performance.

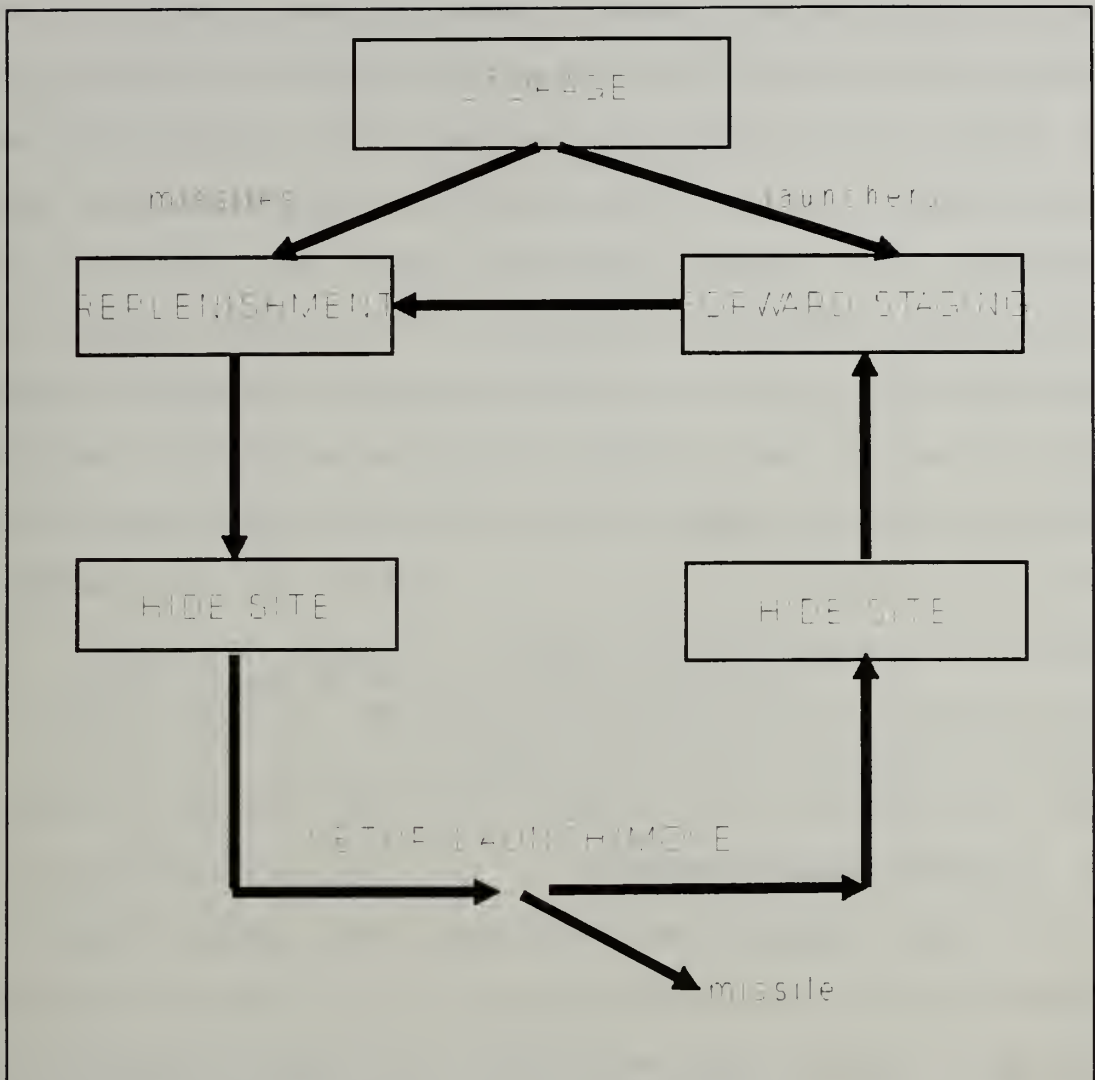
## **B. THE TARGET**

Any effort to quantify search effectiveness must start with the target characteristics. The primary target when looking for TBM launchers is the MAZ-543 transporter-erector-launcher (TEL). It is the primary launch support vehicle for SS-1B (Scud-A), SS-1C (Scud-B), and SS-12 (Scaleboard) surface-to-surface missiles. It is also representative of other surface-to-surface launch support vehicles, and will likely be in service for many years to come. The MAZ-543 TEL is 12 meters long, weighs 29,000 kg (with missile), and is capable of 60 kph on hard surfaces with a range of 1500 km. Typical operations of the TEL are shown in Figure 2. In peacetime the launchers and missiles are located in centralized storage areas. Then sometime during the transition to hostilities the missiles and launchers are

---

<sup>1</sup>See Washburn, A.R., *Search and Detection*, Chapter 5, ORSA Books, 1989 for more information about effort distribution problems.





**Figure 2.** Mobile TBM Launcher Operations.

deployed to Forward Staging and Replenishment sites. Once hostilities commence the launchers operate from their Forward Staging sites, completing a cycle every time a missile is launched. First the TELs will transit to a Replenishment area where a single fueled missile is loaded onto each, then they transit to the launch area utilizing one or more hide sites along the way. At the launch site the missiles are erected, aligned, and fired. The TELs then strike down and transit

back to their Forward Staging area again utilizing one or more hide sites to avoid detection and destruction. The key item to notice in this scenario is that the launchers are not continuously out in the open, able to be detected by enemy sensors. An example of this model is given in the unclassified Warbreaker<sup>2</sup> scenario. In that scenario the transits between the major nodes are several kilometers (about 10-15 minutes), and the time to set-up and strike down from missile launch is about 1 hour and 15 minutes, respectively. That implies that the total time the TEL is out in the open, exposed to surveillance assets, is about 2 hours. The implication of this will be examined in more detail in Chapters III and IV of this thesis.

### C. SEARCHER CHARACTERISTICS

For the purpose of this thesis the searcher will be assumed to be some type of airborne craft and all searchers will have similar characteristics. The specific details of the searcher like its speed, altitude, endurance, and sensor suite will be captured in variables that will then be normalized for analysis. As an example the searcher's detection capability will be characterized by its Sweep Width,

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<sup>2</sup>Warbreaker is a distributed man-in-the-loop simulation developed for the Strategic Defense Initiative Office that has been used to evaluate attack operations against TBM launchers.

W.<sup>3</sup> No effort has been made to compare differing platforms' effectiveness. In general the searcher will have to transit from its base of operations to the search area, search the area with its sensors, investigate possible contacts, and return to its base. One searcher can be relieved on station by another searcher or multiple searchers can search simultaneously to provide greater coverage. The specific details of the searcher, other than it is airborne and that its characteristics can be captured by variables, are not important to this model.

#### **D. SIMILARITIES TO ANTI-SUBMARINE WARFARE**

Countering the TBM problem has been likened to Anti-Submarine Warfare [Ref. 2]. Several key considerations make this comparison valid.

First, both submarines and TBM launchers have a high degree of stealthiness. Both know that to stay alive and to be effective they must use their stealthiness to great advantage. The commanders will choose to give up their stealthiness when it is advantageous to do so. Each must come out into the open to deliver ordnance on target.<sup>4</sup> For the

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<sup>3</sup>See Washburn, A.R., *Search and Detection*, Chapter 4, ORSA Books, 1989 or United States Naval Academy, *Naval Operation Analysis*, 2nd ed., Chapter 6, Naval Institute Press, 1977 for more information about sweep widths and lateral ranges curves.

<sup>4</sup>For the purposes of this discussion it is assumed the submarine will come to periscope depth prior to shooting to verify its target.

opposing force this means that the opportunities for detection will be limited and each opportunity must be capitalized upon.

Secondly, the TBM threat is so complex and the cost of failure so high, that much analysis and effort should go into defeating it. This is similar to the U-boat threat during World War II, and the SSBN threat today. A great deal of analytical thought has gone into fighting submarines from diesel boats of WWII to modern nuclear powered subs of today. Many of the models developed, and particularly the analytic approach to the problem, can be applied to the TBM threat. Ehlers' thesis [Ref. 2] is an excellent example of just such an application. The development of much of Search and Detection theory stems from the genesis of Operations Research itself, finding and countering the U-boats of WWII.

Finally, the operations of WWII U-boats and TBM launchers are very similar. Both are exposed for periods of time where they are susceptible to detection (U-boats proceeding to patrol, TBM launchers moving to the launch area). Both can evade detection by hiding (submerging for the U-boat) but at a cost in time to reach a weapons delivery position. Then once at the weapons delivery position (convoy for U-boat) they both operate in the open for extended periods (surface attack by WWII U-boats) followed by hiding (submerging) and returning to base. Since the basic operations are so similar, the models for search and detection of submarines can be applied to TBM launchers.

## **E. DIFFERENCES BETWEEN TBM LAUNCHERS AND SUBMARINES**

There are at least two major differences between TBM launchers and submarines. First, for submarines to threaten United States forces they have to operate fairly close to those forces, generally in international waters. In international waters there are no restrictions on search operations. However, the TBM launcher with its long range missiles can threaten U.S. forces from within the territorial limits of its country. Since searching for TBM launchers generally requires flying over the host country, search operations conducted prior to war could violate international law and the sovereignty of the nation. In times of war the searcher is vulnerable to enemy actions. The searcher has to have local air superiority in the search area to be most effective. The use of remotely piloted vehicles (RPVs) or unmanned autonomous vehicles (UAVs) would lessen the danger from enemy action, however without local air superiority, the unmanned vehicles could be shot down with the loss of a valuable search asset. Additionally, if the use of remote ground based sensors is planned, local air superiority is needed to deploy the sensors. In Desert Storm air superiority was gained quickly, but this may not always be the case. Searching for the TBM launchers should not be discredited because of the need for air superiority since early control of the air space is a necessity for any campaign to be successful.



Another major difference between searching for submarines and searching for TBM launchers is the detection mechanisms. Submarine detection models based on acoustic detection in open ocean clearly do not apply to the TBM threat.<sup>5</sup> Similarly, models that assume a continuous detection possibility cannot be used directly because of the exposed/hiding nature of the TBM launchers. This does not invalidate all Anti-Submarine Warfare detection models. The analyst however does need to look at the underlying assumptions and determine whether they are valid. Again it is the analytical process that characterizes ASW analysis, not a particular model.

The final difference between submarines and TBM launchers has to do with group operations. Submarines today generally operate independently, and even the U-boats of WWII transited independently. However, the components that make up the TBM force may be highly coordinated. One likely consideration is a salvo. A salvo is the simultaneous launch of many missiles at the same target to try and overwhelm the active defenses. Additionally, since a country is likely to have only tens to hundreds of missiles, centralized control could be common. With centralized control one would expect to see some sort of coordination between the various TBM forces, including either a propensity to shoot missiles at certain times of day or a coordinated deception plan with dummy missiles and launchers.

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<sup>5</sup>This assumes no breakthroughs in detection of TBM launchers utilizing seismic sensors.

Even though submarines and TBM launchers have many things in common they also have some major differences. These differences do not invalidate the comparison of TBM forces to submarine forces, as long as the goal of applying the analytical process is kept in mind.

### III. METHODOLOGY

#### A. BASIC APPROACH

One aspect of Anti-Submarine Warfare that has received much attention is the development of detection models. These models have been used to identify efficient search strategies, to identify which components provide the greatest marginal improvement in performance, and to determine how long to search to achieve specific goals. Similar types of information are required to successfully counter the theater ballistic missile in future conflicts. The models developed to describe searches for submarines can be used, with some modification, to describe searches for theater ballistic missile launchers and support equipment.

One of the most common search models is the random search. In this search model one searcher is trying to find a single target, the target is continuously exposed to detection, and the target is assumed to be stationary. If the searcher has a significant speed advantage over the target (such as airplanes searching for surfaced submarines) the target need not be absolutely stationary for the stationary target assumption to still be valid. However, in the TBM problem the target is definitely not continuously exposed to detection, so any ASW search model has to be modified before it can be



applied to the TBM problem. In practice the random search model closely describes real world search efforts and so the study of the random search model is emphasized. The details of the random search model are presented in section B to lay the ground work for further development.

## **B. RANDOM SEARCH MODEL**

The random search model assumes that the target is randomly distributed in an area A, that the search for the target is conducted in a random manner, that the target is continuously exposed to detection, and that the target is stationary. These assumptions mean that the random search is a lower bound on search effectiveness<sup>6</sup> since a search that is conducted systematically has little or no duplication of effort and is more effective.

The randomness of the search should not be considered an unrealistic approach since even a carefully planned exhaustive search may turn out to be more random than exhaustive in practice. Many things can affect a carefully planned search including navigation errors, gaps in coverage, refueling, investigation of possible contacts, poor turnover between searchers, and other real world events. Likewise the stationary target assumption is not a serious restriction if the searcher has a significant speed advantage over the

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<sup>6</sup>This assumes one does not deliberately try to conduct a more inefficient search by not moving or by not looking.

target. This is because the target cannot move very far during the course of the search. However, target motion does make the search more random in nature. Finally, the continuous exposure assumption does not hold for TBM launcher searches since they may remain hidden for significant periods of time. The random search model has to be modified to account for this fact.

Appendix A gives the derivation of the mathematical formulation for the random search model. The basic formulation is

$$\begin{aligned} P\{\text{det}_{\text{search}}\} &= 1 - e^{-WL/A} \\ &= 1 - e^{-z} \end{aligned}$$

where

$P\{\text{det}_{\text{search}}\}$  is the probability of detecting the target during a search of total length  $L$ ,

$W$  is the sweep width, a term that accounts for the effectiveness of the searcher's sensor in the given environment against the given target,

$L$  is the length of the random search in area  $A$ , and is equivalent to the searchers velocity  $V$  times the total time  $T$  searched,

$A$  is the area being searched that contains the target,

$z (= WL/A)$  is called the coverage ratio [Ref. 8:p. 2-5]. It is a measure of how many times the search area  $A$  is covered in a search of total length  $L$  or time  $T$ , since  $L=VT$ .

Figure 3 shows how the probability of detecting a continuously exposed target varies with the coverage ratio.

As can be seen, a  $z$  of about five all but guarantees that

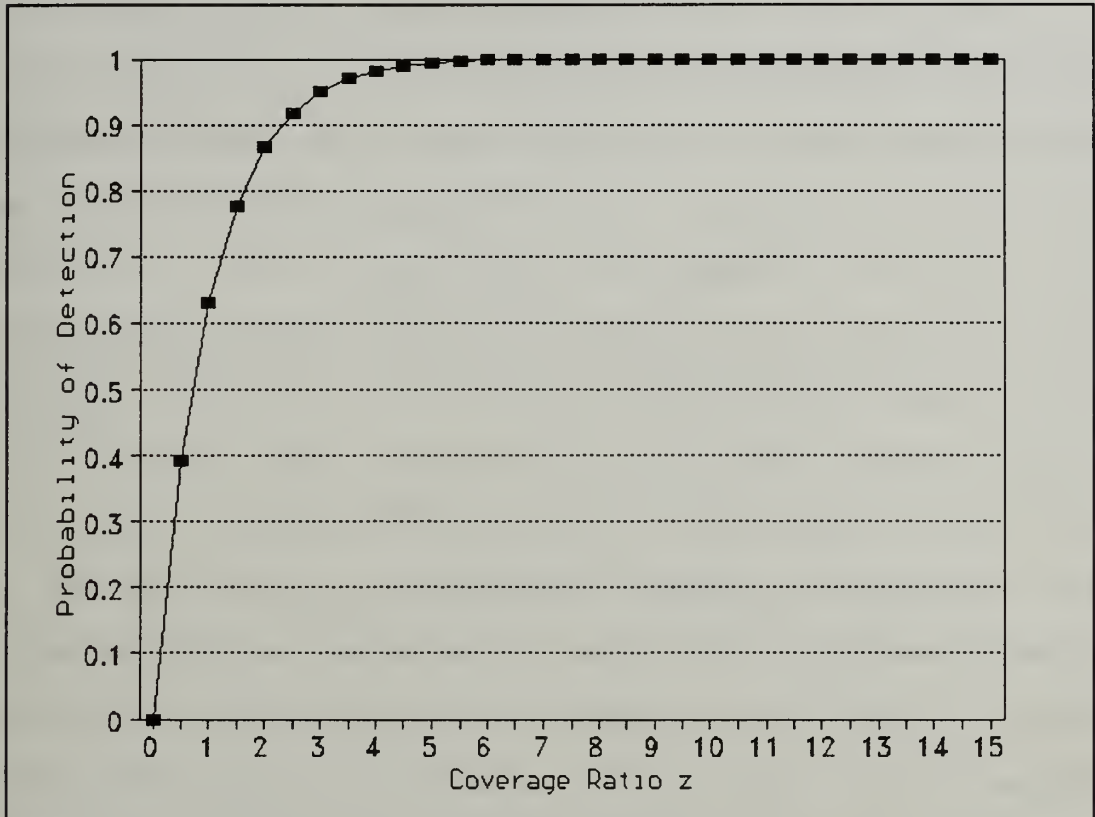


Figure 3. Random Search Model Results.

the target will be detected at least once. Depending on the particular values of the sweep width  $W$ , the area  $A$ , and the searcher speed  $V$ ; the time required to search to achieve any particular  $P\{\text{det}\}$  can be found (choose  $P\{\text{det}\}$ , use figure to get  $z$ , then  $T=zA/WV$ ).

The random search model does not accurately represent the search for TBM launchers since they can be hidden from current

sensors much of the time. To account for this, the random search model must be modified.

### C. MODIFIED RANDOM SEARCH FORMULATION

Consider the case where the target is not continuously exposed to detection. In the derivation of the basic random search model given in Appendix A the first small segment of the search is analyzed. In this segment the development says two things must happen for a detection to occur. The target must be within the searcher's maximum sensor range and the target must be detected. However, in the basic model the target cannot hide; if the searcher gets close enough, detection is guaranteed. This is not necessarily the case for mobile ballistic missile launchers. It is possible the launcher could be stored in a warehouse or hiding under a bridge and not be detectable by search sensors being used. Forrest [Ref. 7:pp. 33-35] accounts for the intermittent detectability of a target by reducing the sweep width. However, most of the factors that affect the target's exposure probability are not controllable by the searcher so we chose to consider the target's actions as a separate term. The classic model was modified to include a third factor for detection to occur; the target must be exposed.

Call the event that the target is physically within the searcher's maximum sensor range  $B$ , the event that the target is exposed  $C$ , and the event that the target is actually

detected D. Then the conditional probability for detection of the target in the small segment of search is

$$\begin{aligned} P\{\text{det}_{\text{segment}}\} &= P\{D|B, C\} P\{C|B\} P\{B\} \\ &= \frac{WL}{NA} P\{C|B\} \end{aligned}$$

Setting  $P\{C|B\}=\phi$  and extending the development of the basic random search model in Appendix A over the entire search, the modified formulation becomes

$$P\{\text{det}_{\text{search}}\} = 1 - e^{-WL\phi/A}$$

Finally, letting  $z = WL/A$  the modified random search model becomes

$$P\{\text{det}_{\text{search}}\} = 1 - e^{-z\phi}.$$

The key factor in this modified random search model is  $\phi$ . It is the probability that the target will be exposed when the searcher is within effective detection range  $W$  of the target. If the target does not know the searcher is present or if the target does not change his operations even if he knows when the searcher is present, then  $\phi$  can be estimated by the proportion of the search time that the target is exposed. For example if the target is exposed on the average two hours per day and the search area is searched continuously 24 hours per day, then  $\phi=2/24=0.083$ .

If the target always knows when the searcher is present and hides, then the probability of detection is zero. But the enemy will never shoot any missiles if the searcher can search



continuously 24 hours per day since the launcher must be exposed to detection to launch.

Search effectiveness by the searcher can be improved if he only searches when the launchers are exposed to detection. By only searching when the launchers are exposed,  $\phi$  approaches one (the random search model). The searcher requires excellent intelligence information to approach this ideal. The type of intelligence information required might be provided by remote ground-based sensors, human spotters, or through analysis of historical operations. In general  $\phi$  can be either deterministic or random and the behavior of this model with respect to  $\phi$  will be explored in the next Chapter.

#### D. EXTENSION TO SIMULTANEOUS MULTIPLE SEARCHERS

Now consider the case where M identical searchers are simultaneously looking for a TBM launcher. Let the  $i$ th searcher have a sweep width of W and total search length L. Then the probability of detecting the TBM launcher in that first segment of search effort for the  $i$ th searcher is the same as the one searcher one target case. However, now the search is simultaneously repeated M times over the total search area A. Therefore the probability of any searcher detecting the target during the search of the first segment is M times the one searcher one target case

$$P\{\text{det}_{\text{segments}}\} = \frac{MWL\phi}{NA}$$

Extending the development as in Appendix A over the entire search of length  $L$ , the probability of any searcher detecting the TBM launcher during the length of the search becomes

$$P(\text{det}_{\text{search}}) = 1 - e^{MWL\Phi/A}$$

If all  $M$  searchers are not identical, and the  $i$ th searcher has a sweep width  $W_i$ , then the probability of any searcher detecting the target in the first simultaneous segment of search is

$$P(\text{det}_{\text{segments}}) = \frac{L\Phi}{NA} \sum_{i=1}^M W_i$$

Over the entire search effort this becomes

$$P(\text{det}_{\text{search}}) = 1 - e^{-(L\Phi \sum_{i=1}^M W_i)/A}$$

where there are  $M$  total searchers, the  $i$ th searcher has a sweep width of  $W_i$ , and each searches simultaneously over a total length  $L$ .

#### IV. ANALYSIS OF MODIFIED RANDOM SEARCH MODEL

In Chapter III the random search model and a modified formulation of the random search model were discussed. The modified random search model incorporated the probability that the target may not be continuously exposed to detection. In this chapter we will look more closely at the probability that the target is exposed to detection and the effects on the overall probability of detecting the target with a given search effort.

##### A. DETERMINISTIC EXPOSURE

The first case to consider when investigating the probability that the target is exposed, is the deterministic case, when  $\phi$  is a constant. In the deterministic problem the probability that a target is exposed to detection when the searcher is within sensor range is fixed (for example, it may be 20 percent so that  $\phi = 0.2$ ). First it must be assumed that the target does not alter his operating patterns due to the search effort. This means either that the target does not know when the searcher is near, or that the target does not care. Secondly it must be assumed that the target operating characteristics do not vary directly with the time of day and that target operations are uniformly distributed around the



clock. With these assumptions the probability of target exposure for any searcher is equal to the overall proportion of time that the target is exposed. If the target, on the average, spends 12 out of every 24 hours exposed then the probability that the target will be exposed for a random searcher is 0.5 (50 percent).

Figure 4 shows how the probability of detection for a random search varies with the coverage ratio ( $z$ ) and the proportion of time the target is exposed ( $\phi$ ).

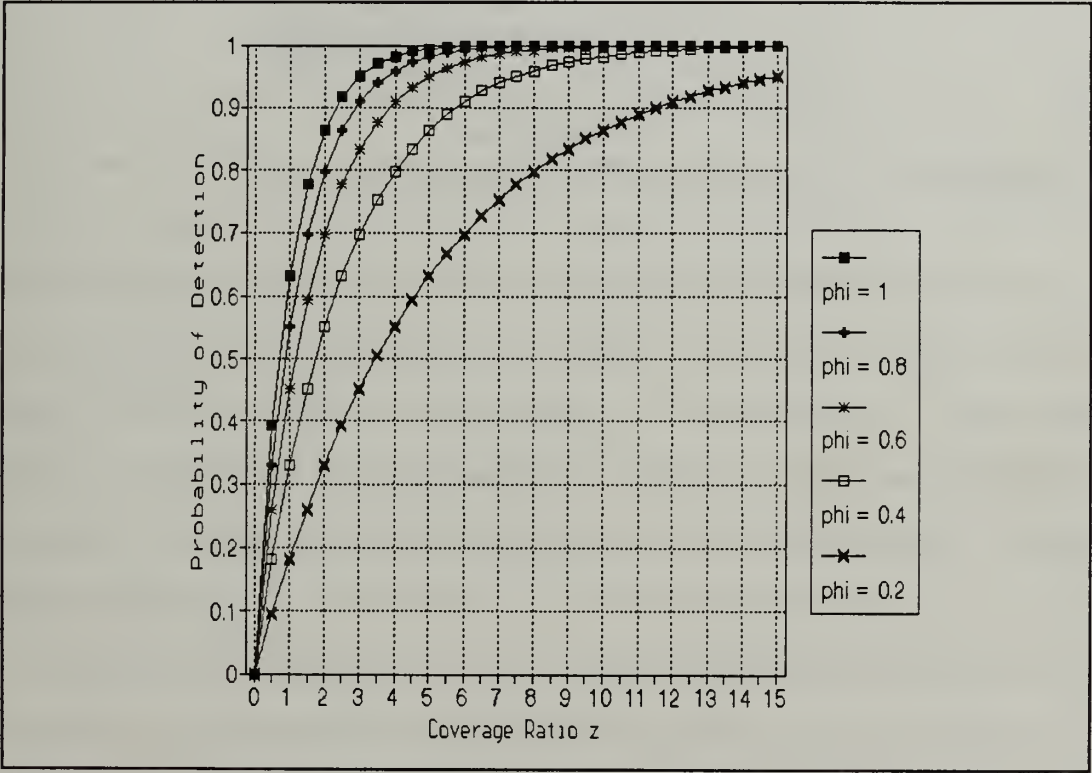


Figure 4. Modified Random Search Model Results.

Figure 5 uses the same data to show the coverage ratio required to achieve various probabilities of detection versus the proportion of time exposed.

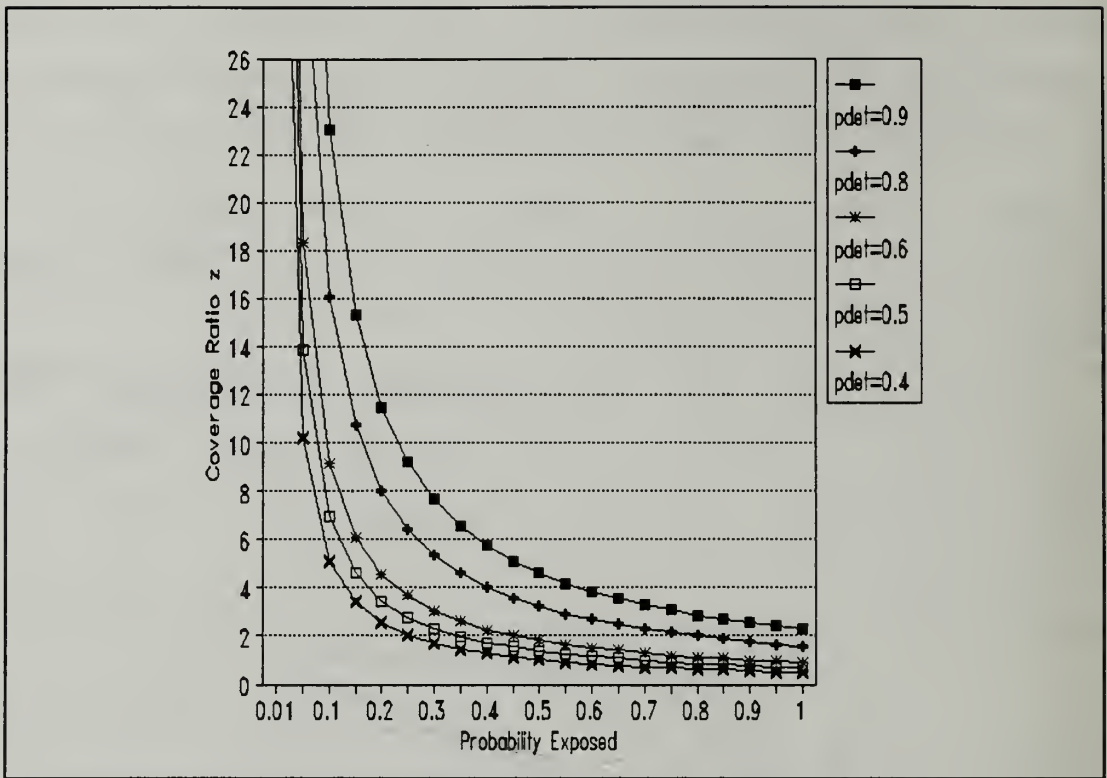


Figure 5. Required Level of Search Effort.

As the proportion of time that the target is exposed falls below about 0.3 or 30 percent the required coverage ratio increases dramatically. For an 80 percent probability of detection one has to cover the search area 16 times in a given search if the target is exposed 10 percent of the time. To achieve the same probability of detection for the same amount of search effort, the search area only has to be covered about 3.5 times if the target is exposed 50 percent of the time. If the target was continuously exposed to detection, as in the classic case, a coverage ratio of less than two is required.

The amount of search effort required to achieve a constant probability of detection is very dependent on the probability

that the target will be exposed when within sensor range of the searcher. Next we will consider the exposure probability to be a random variable and analyze the results.

## **B. UNIFORMLY DISTRIBUTED EXPOSURE**

The required search effort for a desired probability of detection as measured by the coverage ratio is extremely dependent on the proportion of time the target is exposed to detection. In the last section it was assumed that the target exposure proportion was deterministic. However, in real world situations, without good intelligence information one is unlikely to know very much about the target operations. This will most likely be true in the first several days of a conflict when no operational data has yet been obtained. Additionally, for any particular search mission, the target may be exposed a random fraction of the search time even if overall the exposure proportion is constant. For these reasons we analyzed the level of effort required when the probability that the target is exposed to a searcher is itself a random variable.

The first random variable used to characterize target exposure was the uniform random variable. The probability of target exposure is assumed to vary between zero and one with equal likelihood and the expected value is one-half. The uniform random variable is used when one knows little or nothing about the true probability of the target being exposed

to detection. Again this is probably a good assumption when looking at a single search mission provided the target does not change his operating characteristics due to the search effort. Since the probability of the target being exposed in the small element of search is now a random variable, the overall probability of detection for the search also becomes a random variable. By assuming a uniform probability distribution for the target exposure, an unconditional probability of detection can be determined that is a function of the coverage ratio. Appendix B gives the details of computing the unconditional probability of detection for the search when the target exposure  $\phi$  is uniformly distributed between zero and one. The result is that

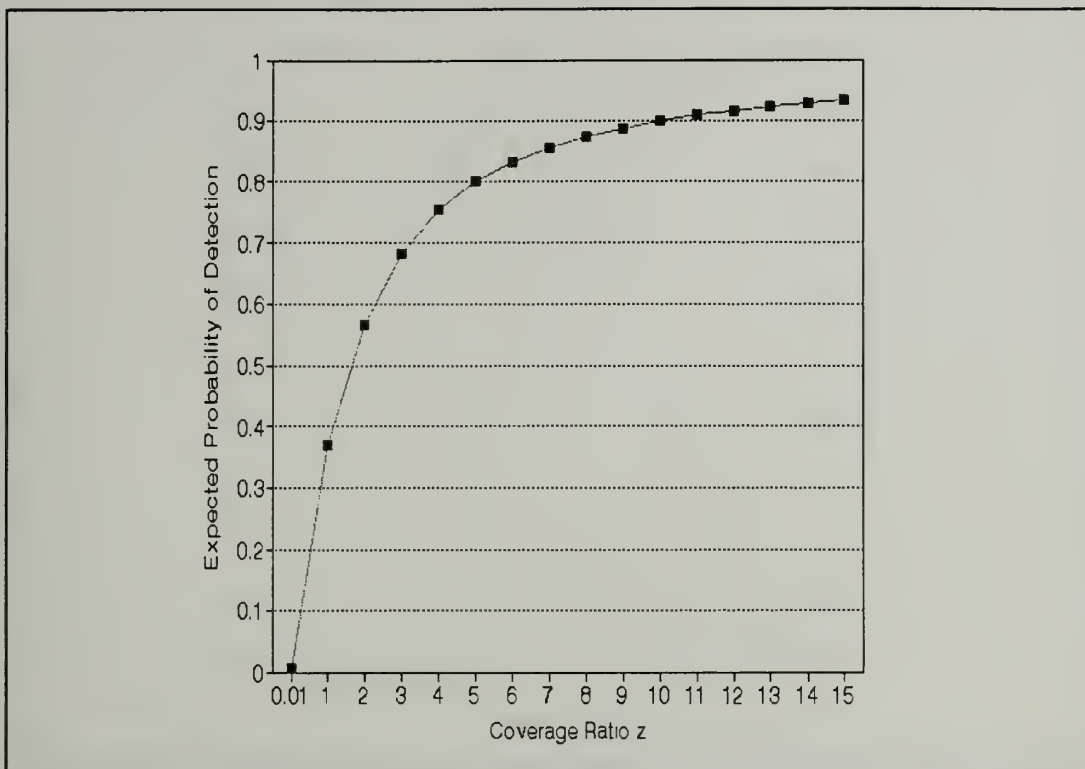
$$P\{\text{det}_{search}\} = 1 - \frac{1}{Z} + \frac{e^{-Z}}{Z}$$

Figure 6 shows how the unconditional probability of detection varies with the coverage ratio. For an 80 percent probability of detection overall, a coverage ratio of about five is required. This corresponds roughly to a deterministic value of  $\phi$  of about 0.4.

### C. TRIANGULARLY DISTRIBUTED TARGET EXPOSURES

#### 1. Triangular Distribution with Mean of One-half

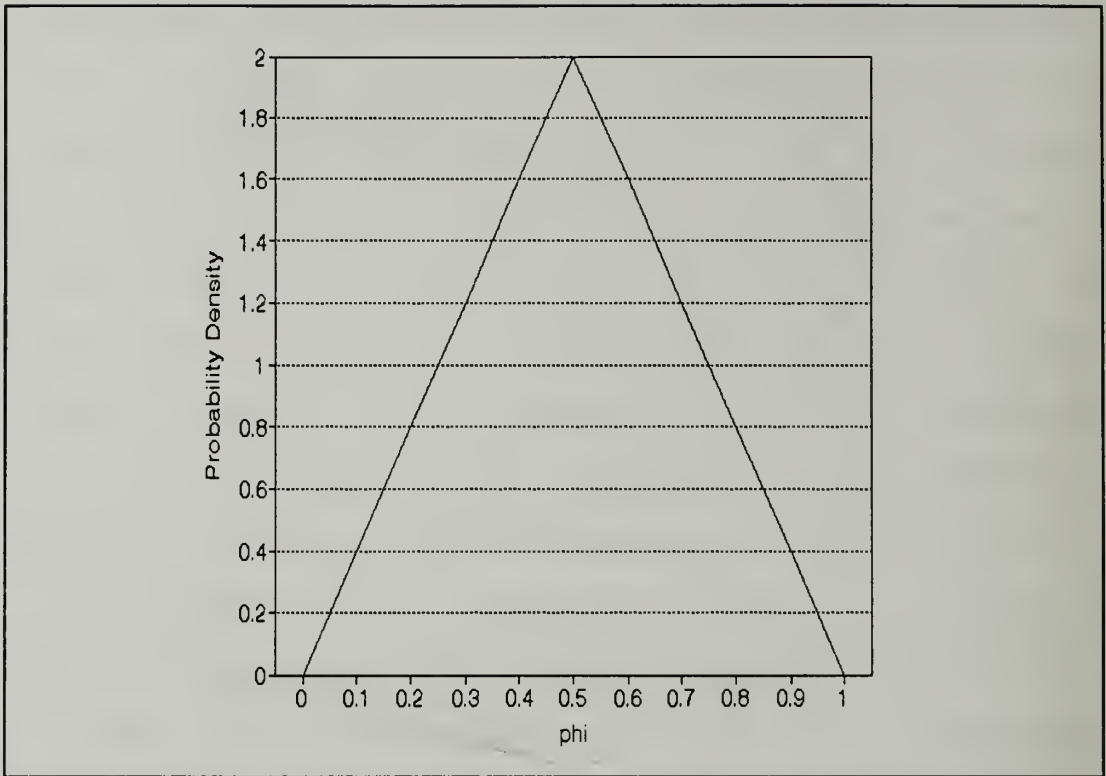
In this section the probability that the target is exposed when the searcher is within maximum sensor range is again assumed to be a random variable. However this time a



**Figure 6.**  $\Phi$  Distributed Uniform (0,1).

triangular distribution with a mean and a mode of 0.5 is assumed. The probability density function for this triangular distribution is shown in Figure 7. This type of distribution corresponds to the case where some reaction to the search effort is expected from the target. Sometimes, the target knows when the searcher is overhead, so it is only exposed on the average half of the time. But now the variability around that average value is less than the uniform case. It is unlikely that the target will be continuously exposed or that the target will never be exposed at all. The probability of detection for the search is a random variable conditioned on the value of  $\phi$  and Appendix B has the details of the





**Figure 7.** Density of Triangle Distribution.

derivation of the unconditional probability of detection

$$P\{\text{det}_{\text{search}}\} = \frac{1}{z^2} (8e^{-\frac{z}{2}} - 4e^{-z} - z^2 - 4)$$

Figure 8 shows how the unconditional probability of detection for the search varies with the coverage ratio when  $\Phi$  is distributed triangularly. Note that now for an 80 percent unconditional probability of detection that a coverage ratio of less than four is required. The results are more favorable to the searcher when  $\Phi$  is distributed triangularly than when  $\Phi$  is distributed uniformly, even though they both have the same expected value of one-half. This is because the triangular distribution has less variance about the mean.



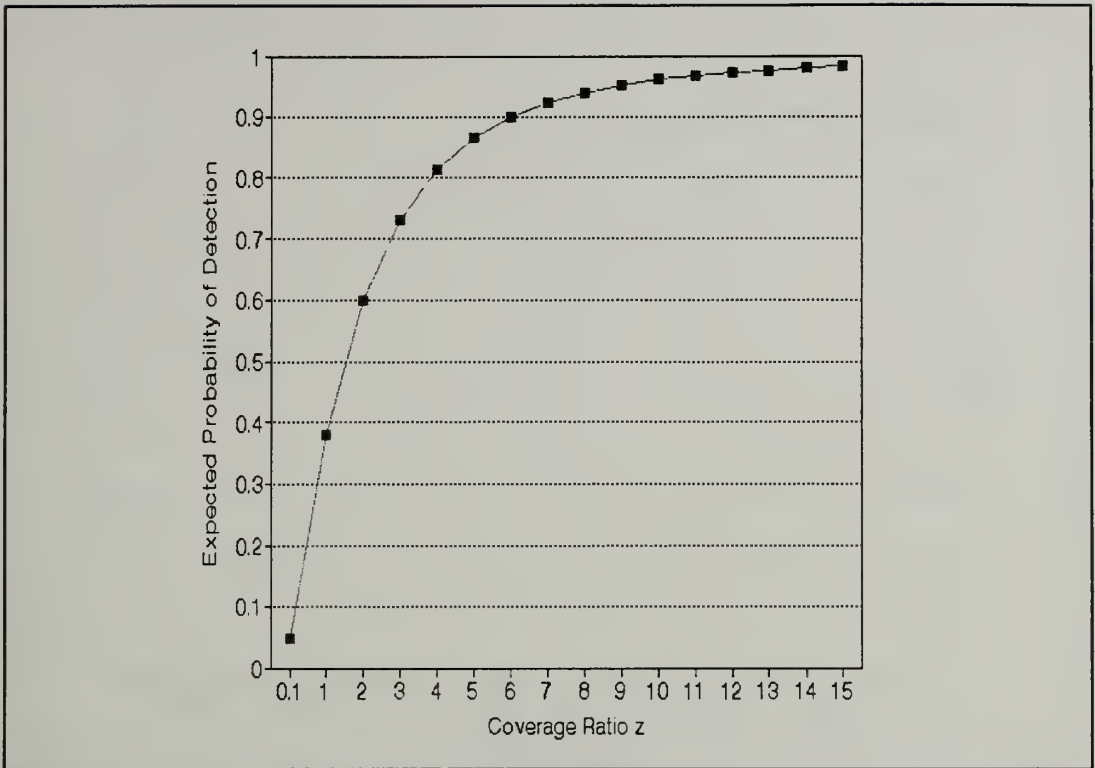


Figure 8.  $\Phi$  Distributed Triangularly with Mean One-half.

## 2. Triangular Distribution with Mean of One-Third

Next consider another triangle distribution. This time consider the distribution shown in Figure 9. The distribution has a mean of one-third and a mode of zero. This distribution is indicative of a target that has a good idea of when the searcher is near so it is hidden most of the time. Another interpretation is that the target is only exposed a small fraction of the search time because the nature of the operations only require relatively short exposures. The actual probability that the target is exposed varies, but it is heavily weighted toward no exposure with no chance of being exposed for the entire search effort. Appendix B has the

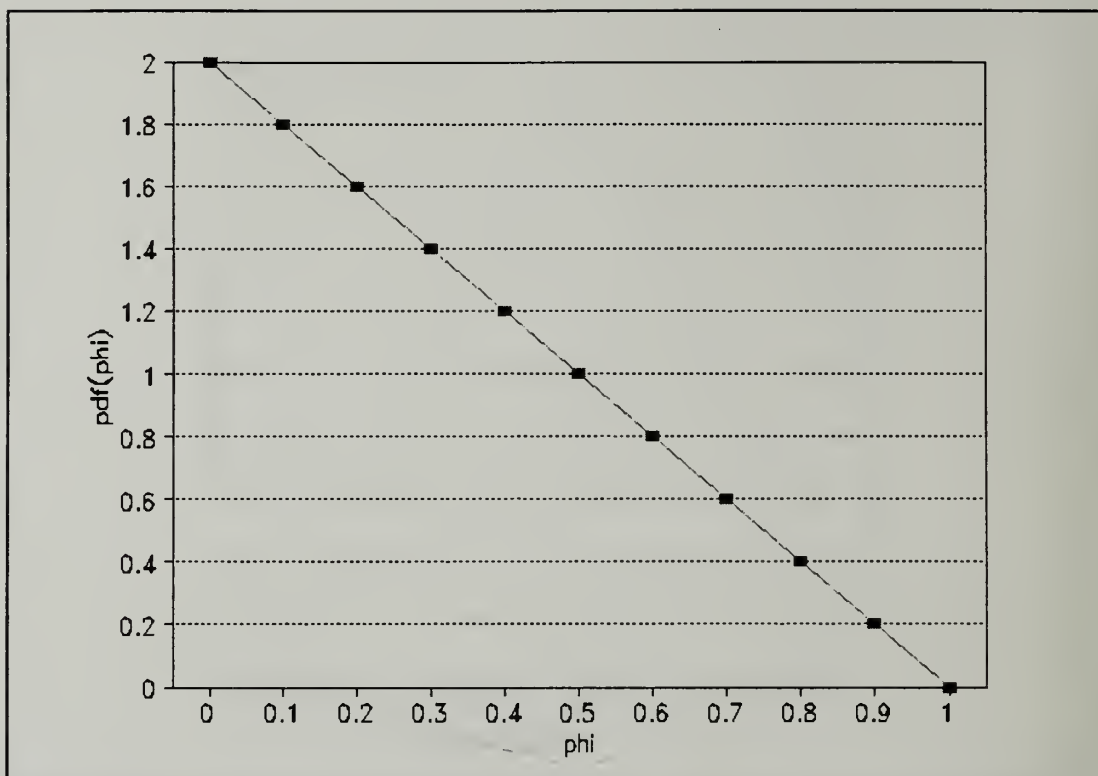


Figure 9. Density of Triangle Distribution Mean One-Third.

derivation of the unconditional probability of detection. The formula for the unconditional probability of detection as a function of the coverage ratio is

$$P\{\text{det}_{\text{search}}\} = \frac{1}{z^2} (z^2 - 2e^{-z} - 2z + 2)$$

The unconditional probability of detection is plotted versus the coverage ratio in Figure 10. For an 80 percent probability of detection a coverage ratio of about nine is required. This compares to a coverage ratio of about four when  $\Phi$  is distributed triangularly with mean one-half and a coverage ratio of just over 1.5 when the target is continuously exposed.

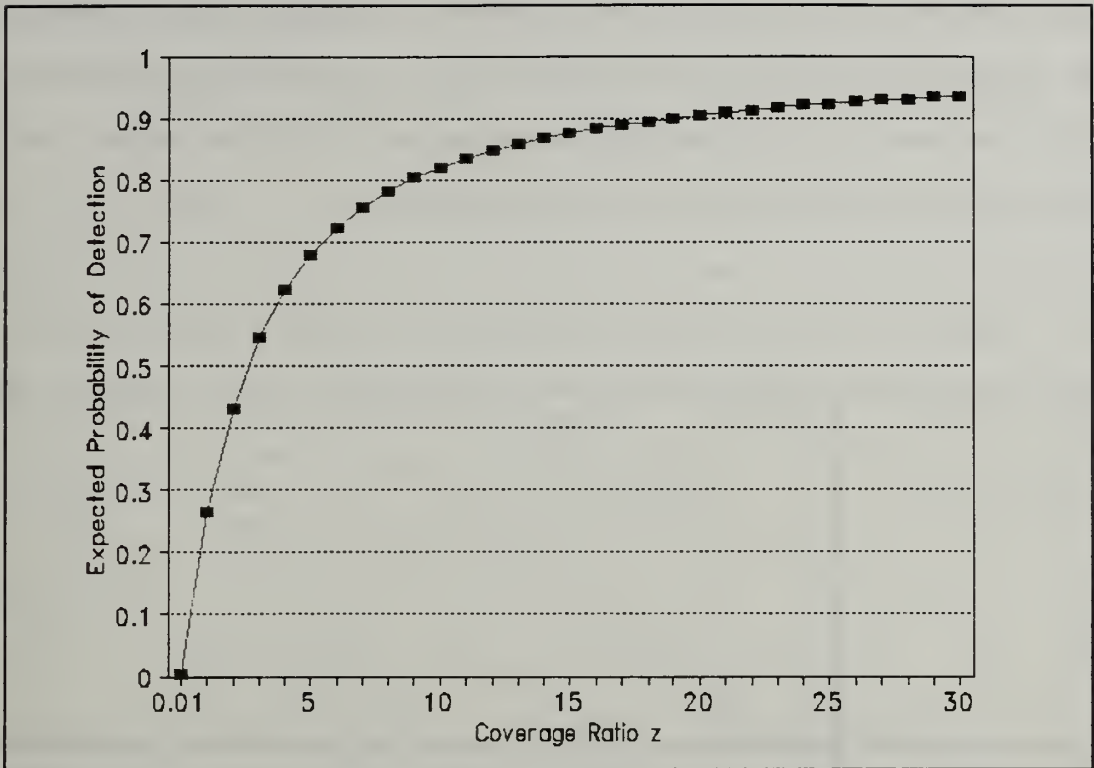


Figure 10.  $\Phi$  Distributed Triangularly with Mean One-Third.

#### D. SUMMARY

In this chapter we looked at how the probability of detecting an intermittently exposed target during a search varies with the coverage ratio of the search. We examined several cases, both deterministic and stochastic, for the probability that the target is exposed when the searcher is within sensor range. Specifically, the probability of the target being exposed was considered to be deterministic, Uniform (0,1), Triangular (0,1) with mean one-half, and Triangular (0,1) with mean one-third. Table 1 summarizes the results. It can be seen that the amount of search effort, as

measured by the coverage ratio, varies significantly depending on whether the target is continuously exposed to detection, is only exposed to detection a fraction of the time, or is exposed a random fraction of the time.

TABLE 1. COMPARISON OF REQUIRED COVERAGE RATIOS.

Prob. of Detection	Phi = 1.0	Expected Value 1/2			Phi Triangle (0,1) mean 1/3
		Phi =	Phi Triangle (0,1) mean 1/2	Phi Uniform (0,1) mean 1/2	
		0.5			
0.9	2.4	4.5	6	10	19
0.9	1.6	3.2	3.9	5	9
0.5	0.8	1.2	1.5	1.8	2.5
Variance $\Phi$	0	0	1/24	1/12	1/18

The coverage ratio required for a given probability of detection is a function of the average exposure proportion as well as the variance of that exposure. First, three cases with mean exposures of one, one-half, and one-third are shown. The lower the mean exposure the higher the required coverage ratio for any desired probability of detection. The increase in required search effort is not linear and as was shown in

Figure 5, the required coverage ratio increases dramatically for mean exposures below about one-third. Next, compare the three cases where the expected target exposure is one-half to see the effect of the variance of  $\Phi$  on the results. The deterministic case has no variance and has the lowest required coverage ratio for any desired probability of detection. For the probabilistic distributions the required coverage ratio increases with increasing variance for all probabilities of detection. The required search effort to achieve a certain probability of detection increases with lowering expected exposure and increases with increasing variance. The next chapter will explore how to achieve these required coverage ratios including the implications to TBM counter-force campaign planning.

## V. TACTICAL IMPLICATIONS OF SEARCH MODEL

In the last chapter the results of the random search model and the modified model were presented. In this chapter we will interpret the results in terms of the physical world and attempt to show the implications of the model on campaign planning and future research and development efforts.

The modified random search model states that the probability of detecting a target during a search is an exponential function that depends on the sweep width of the searcher  $W$ , the probability that the target is exposed when within sensor range of the searcher  $\phi$ , the size of the area being searched  $A$ , and the total length searched  $L$ .

$$P(\text{det}_{\text{search}} | \Phi = \phi) = 1 - e^{\frac{-WL\phi}{A}}$$

Further, in the last chapter we showed how the probability of detection depends on the mean and variance of the probability of the target being exposed  $\phi$ .

The purpose of the attack operations against the TBM launchers and support equipment is to reduce or eliminate ballistic missile launches. This reduces the number of missiles that have to be countered by active defenses, thus making them more effective. Additionally, the attack operations can reduce the possibility of large salvos of



missiles that could saturate the active defenses. Therefore, attack operations can be effective even without destroying ballistic missile launchers, if the operations result in fewer and less frequent missile launches. This chapter will discuss ways that attack operations can be improved by considering factors that enter into the search model and considering some factors that are outside of the model.

#### **A. FACTORS AFFECTING THE COVERAGE RATIO**

The coverage ratio  $z = WL/A$  is a measure of the level of search effort required to achieve a certain probability of detection. The coverage ratio is the fraction of the total area covered by the searcher during a search effort that covers a total distance  $L$  and width  $W$ . Since in a random search the formula is exponential some locations are covered more than once during the search. We saw in the last section that to get a specific probability of detection a certain minimum coverage ratio was needed. Ways to achieve greater coverage ratios will be examined by looking at the factors that contribute to the coverage ratio.

##### **1. Sweep Width**

The sweep width is a measure of the effectiveness of the searcher to find the target in a given environment. The sweep width is a function of many things including the type of sensor used, the speed and altitude of the searcher, the alertness of the sensor operator, the environment, the time of

day, the terrain, etc. In general, the larger the sweep width the larger the coverage ratio for any length of search effort. However the sweep width as used here refers to an effective sweep width, that is, the range such that a target can be identified as a TBM launcher. Because of terrain and environmental considerations, just increasing the range of a radar may not increase the sweep width, unless the increased range includes increased identification ability. Most of the factors that affect the sweep width are outside of the control of the searcher, but a few are not. First, the searcher should use the most effective sensors possible. This also means the planner should consider which platforms to assign to particular missions. For example a night mission should be flown by aircraft with infrared or low-light-level sensors as well as radar. The next factor is searcher speed and altitude. Generally for current sensors the lower and the slower an aircraft flies the better the effective sweep width of all sensors but the greater the danger from enemy action. Items for future development that can improve the sweep width include better sensors, manned aircraft that are less susceptible to enemy fire (stealthy or good defenses), and remotely or autonomously piloted vehicles.

## **2. Total Search Length**

The total search length  $L$  is a direct measure of the amount of time spent searching since time equals the total

length divided by the searcher's speed. The longer one searches the higher the probability of detecting the target. Therefore one way to increase the coverage ratio is to search longer, where continuous coverage 24 hours a day is the most one can do. If the search assets are available to provide continuous coverage for the length of the campaign, then any further improvements in search effort must be made by increasing the sweep width or speed of the searcher. For example if the searcher flies at 400 nm/hr and the sweep width is 1 nm, then continuous coverage yields 9,600 sq nm covered per day. If the search area is 10,000 sq nm then the coverage ratio is about one per day and to get a required coverage ratio of ten would require 10 days of search. With most sensors there is an optimum speed. Searching too fast reduces the sweep width and searching too slow doesn't cover enough ground. So another way to increase the coverage ratio is to search longer or to change the search speed closer to the optimum value.

### **3. Search Area**

The last parameter in the random search equation that the operator can affect is the size of the search area. By using programs like Lt. Hair's [Ref. 6] that identify areas in which the TBM launchers are likely to operate the size of the search area can be reduced. The factors that influence these programs can include terrain data, likely ballistic missile

aim points, ballistic missile range, command and control nodes, and transportation networks. If the search area is reduced too much though it may no longer contain the target and no amount of search effort will find it. Therefore the search area must be sufficiently large to reasonably contain the target but should not be excessive. If the target can hide then the use of negative search information to reduce the search area has limited value. One has to be careful to say the target is not in an area because it wasn't detected, since it is possible the target was not exposed to detection during the search.

## **B. FACTORS AFFECTING TARGET EXPOSURE TO DETECTION**

Chapter IV showed that the level of search effort to achieve a set probability of detection is highly dependent on the probability that the target is exposed to detection when the target is within the searcher's sensor range. The probability of target exposure is mostly a function of the target's actions and priorities, but there are a few things the searcher can do. The more the target is exposed the better the chance the searcher can find him. The following sections discuss tactical factors that can affect the likelihood of the target being exposed to detection.

### **1. Use of Passive Sensors**

The first tactic the searcher can use to keep the target exposed is to use passive sensors such as visual and

infrared. If the target does not know when the searcher is near he cannot avoid detection by hiding. However, the reduced sweep width of those sensors may be a factor. The U.S. Navy used active sensors to try to find submarines for years with little success because the submarines could detect the searchers long before they could hope to detect the submarine. The submarines would just avoid the searcher by staying hidden, the same is true for the TBM launcher forces.

A very important example of this tradeoff between target exposure and effective sweep width follows. If by staying passive the probability of the target being exposed goes from 0.2 to 0.4, Figure 5 shows that for an 80 percent probability of detection, the coverage ratio goes from about eight to about four. This is a **50 percent** reduction in the required search effort and probably would offset the reduced sweep width of passive sensors.

## **2. Use of Active Sensors**

The use of active sensors can also be beneficial. Since the measure of effectiveness of attack operations is reducing the number of missiles launched, flooding an area with active emissions can cause the TBM force to remain hidden. The probability of detecting the launchers will be low but the operation will be a success because no missiles will be launched. It must be assumed that any country that can develop a ballistic missile capability can also develop



the capability to detect radar emissions (electronic surveillance measures, ESM). However, for the active sensors to be a deterrent to operations, the threat must be credible; that is the active sensors must be able to detect the launchers. At some point the need to conduct their mission will force the TBM forces to expose themselves. If the launchers are not detected and prosecuted, the TBM force will realize the threat is not credible and active sensors will no longer be a deterrent. Active sensors can reduce the number of missile launches and prevent salvos of missiles from being launched if the TBM forces can detect the active sensors and if the use of active sensors represents a credible threat to TBM forces.

### **3. Use of Area Denial Weapons**

A second tactic that can be used to increase the target's exposure is to use area denial weapons like land mines in likely operating areas. Even if the target does not hit a land mine, it will probably have to slow down to make sure it avoids the mines. By reducing the target's effective speed, its' exposure time is increased.

### **4. Destruction of Roads and Bridges**

Another tactic would be to randomly bomb roads and bridges. It is unlikely that the launchers will be hit, but they will be forced to go around damaged areas. When



avoiding damage to improved roads the launchers will have to go slower and will be more exposed to detection.

### **C. MULTIPLE SEARCHERS**

Another method to achieve the required probability of detecting the target is to use multiple searchers. As was shown in the last chapter, multiple searchers add to the amount of area being swept per unit time. Two identical searchers operating simultaneously will sweep out more search area than one searcher. However, adding more searchers to reduce the expected time till detection may not be valid due to the random nature of the probability of target exposure. For example, if a coverage ratio of ten is needed for 80 percent probability of detection and one searcher sweeps out one coverage ratio per hour, then ten identical searchers should achieve the 80 percent value in one hour. However, the probability of target exposure values are long run averages and will probably not hold over the relatively short period of one hour. But ten searchers operating over many hours should perform like the model predicts, discounting non-random events.

### **D. FACTORS OUTSIDE OF THE SEARCH MODEL**

The final factors to be consider are the ones that are not explicitly stated in the search model. Factors like training, tactics, and the use of operational analysis are not part of

this model, but can offer significant improvements in search performance.

## **1. Historical Analysis of Operations**

Another way to improve search performance is through historical analysis of target operations. This is an area that the Anti-Submarine Warfare people have long stressed. By looking at the operations of the target over several days or weeks, patterns will emerge that can be exploited. For example over 80 percent of the TBM launches during the Persian Gulf war were at night [Ref. 3]. Patterns like this allow the searcher to tune the search effort. By searching when the launchers are historically active, the probability that a target will be exposed to detection during the search increases and so does the probability of detecting the target.

## **2. Cuing the Search Effort**

A second factor to be considered is cued search. If the searcher could know when the target is exposed to detection, the searcher wouldn't waste search effort when the target was hiding. Tuning the search effort to when it is most beneficial effectively increases the probability of the target being exposed during the search and increases the probability of detection. One way to know when the target is exposed is to use remote ground based sensors that can detect when a launcher is moving. Such sensors were used during the Vietnam War to indicate troop and resupply movements.

Additionally, human intelligence such as observers in place behind enemy lines could perform the same mission but with a much higher risk. Having observers on the ground, either human or electronic can provide the searcher with detailed information about when and where targets are exposed to detection. Using such information can significantly increase the chances of detecting the target.

### **3. Training and Practice**

Having a warfare community that is trained in search theory and routinely practices searching for TBM launchers can greatly improve search effectiveness. Another good quality of the Navy Anti-Submarine Warfare community is the emphasis on training and practice. Searching for TBM launchers requires specialized training in search theory, target recognition, and search tactics. These items must be included into the training of all attack aircraft crews. Additionally, the attack operations mission has to be specifically assigned to a few squadrons which can emphasize this type of training. Along with practice and training goes the evaluation of effective tactics and the evaluation of new sensors and weapons. As an additional benefit these squadrons will become repositories of knowledge about how best to counter the TBM launchers. To improve performance in the attack operations mission, attack squadrons must train to counter TBM launchers. Additionally, a few squadrons should specialize in these

counter-force operations to develop and evaluate new tactics, to evaluate new sensors and weapons, and to maintain a knowledge base about countering the TBM threat.

#### **4. Destroy or Trail**

One final way to effectively improve search performance is to follow a TBM launcher once it is detected. By following a launcher it may be possible to identify launcher or missile storage areas or to further refine the operating areas and patterns. However if the launcher is subsequently lost nothing will have been gained. Indeed Decision Theory suggests that if the expected benefit from following the launcher in terms of additional launchers killed is less than one, then the launcher should not be followed and it should be attacked. Launchers can also be marked for easier trailing. Methods of marking a launcher can include florescent paints, or paints with radioactive elements that can be applied from the air. Ideally, if the goal is to follow the launcher then the paints could be applied across a roadway ahead of the launcher which would then be picked up on the wheels or tracks of the launcher. This way the launcher would be identified for easier tracking without the launcher crew knowing and reacting. Other methods of identifying and marking TBM launchers can be developed and evaluated. However for any of these methods to be worthwhile the probability of losing the launcher must be fairly low,

otherwise the searcher would be better off, in terms of numbers of launchers destroyed, attacking and destroying all launchers that are detected.

#### **E. SUMMARY**

In this chapter the various factors that affect search performance were discussed. Those factors were divided into four major categories. First the explicit model parameters were discussed indicating the types of actions that could improve search performance. The explicit model parameters discussed included terms in the coverage ratio and the probability of target exposure. Next the use of several searchers working simultaneously was discussed as a way to cover more of the search area in a limited amount of time. Finally, factors not explicitly detailed in the model were discussed. The performance of search assets in the Persian Gulf War was poor but by analyzing search models like this one systematic improvements in performance are possible.



## VI. SUMMARY AND CONCLUSIONS

### A. SUMMARY

In this thesis we have seen that search models originally developed to locate WWII U-boats can be applied, if appropriately modified, to the analysis of modern tactical ballistic missile searches. The appropriate modification to the random search model was to include an additional parameter that accounts for the fact that TBM launchers can be hidden from view when the searcher is within sensor range. This type of modified model can also be used for modern diesel submarine warfare in shallow water where acoustic detection is unlikely and the diesel submarine can hide by submerging or lowering its periscope. A model developed for use against diesel submarines (really surface ships that could submerge) has been updated and fifty years later can be used against diesel submarines and TBM launchers.

When analyzing the results of the modified search model it became apparent that a target that can hide can significantly increase the search effort required to achieve the same probability of detection. A target that is continuously exposed requires a coverage ratio of about two for an 80 percent probability of detection, while the same target if only exposed a constant 20 percent of the time requires a



four-fold increase in the coverage ratio to about eight to achieve the same 80 percent probability of detection. We also considered the possibility that the probability of target exposure could be a random variable. For the three distributions considered for the random variable, the required search effort increased with decreasing mean exposure and increased with increasing variance. Depending on the desired probability of detection, the required search effort increased by a factor of eight or more.

As the results of Desert Storm showed, finding mobile TBM launchers is a difficult problem. This modified random search model illustrates the additional search effort required to find a non-cooperative target. The model also highlights how the entire search effort is strongly affected by the tactics used by the mobile TBM launcher crews. To improve the performance of the United States military against mobile TBM launchers requires a combination of training with existing systems and the procurement of very capable systems in the future. Models like this allow the analyst to properly evaluate the conflicting requirements.

## **B. CONCLUSIONS**

The most efficient way to reduce the number of ballistic missiles reaching a target area is to destroy the fixed and mobile TBM launchers and support equipment. By removing an enemy's means of launching and supporting ballistic missiles,

we greatly reduce the threat to friendly forces. Additionally, a campaign of attacking TBM launchers increases the effectiveness of active defensive systems because they are less likely to be overwhelmed by large salvos of missiles.

To be successful at attacking mobile ballistic missile launchers prior to launch, improvements must be made in several key areas. This model suggests several ways to improve pre-launch tactical ballistic missile launcher detection. Those improvements include:

- developing better sensors, preferably passive, which allow for the detection of TBM launchers at greater ranges
- increasing the time-on-station capabilities of search assets
- reducing the area to be searched by using computer aided search models to identify likely operating areas
- using area denial weapons like mines to increase the amount of time the launchers are exposed to detection
- destroying roads and overpasses in TBM launcher operating areas to increase the exposure time
- developing and using remote ground based sensors or human observers to better refine when and where to search
- increasing the number of searchers, which increases the level of effort
- using analysis of historical operations to refine when and where to search
- marking or trailing launchers once they are found if the expected benefit of finding a staging area is greater than the benefit of killing that launcher
- developing search tactics that make the search effort less random and more systematic

These suggestions can greatly improve the performance of friendly forces in reducing the threat from Tactical Ballistic Missiles.

### C. POSSIBLE FOLLOW-ON RESEARCH

With the increased proliferation of tactical ballistic missiles, the application of search theory to locate TBM launchers will become increasingly important. Additionally, as the Defense Department downsizes, there will be increased emphasis on simulations to evaluate new tactics. The modified random search model presented in this thesis is relatively simplistic. It only models the search performance of simultaneous, identical searchers against a single target. A more general formulation that allows for a time varying number of searchers and targets, each with different characteristics is possible for computer implementation.

Let  $m(t)$  be the number of searchers at time  $t$ , and let  $n(t)$  be the number of targets at time  $t$ . Then the detection rate at time  $t$  is given by

$$\gamma(t) = \frac{V W m(t) n(t)}{A}$$

where  $V$  is the searcher velocity and  $W$  is the searcher sweep width and  $A$  is the search area. The probability of detecting any target by time  $t$  is

$$P_{\text{det}}(t) = 1 - e^{-\int_0^t \gamma(\tau) d\tau}$$

In a computer implementation, the entire detection rate can be replaced by an  $m$  by  $n$  matrix where each element represents the detection rate of searcher  $i$  against target  $j$ . This allows for each searcher to have a different detection rate for each target. The total detection rate can be computed by multiplying the detection rate matrix on the left by the vector of searchers and on the right by the vector of targets. Such a computer implementation of the random search model would be more robust than the mathematical model presented in this thesis. Follow-on research could identify specific scenarios like a large first strike or a salvo of five missile launches. From these likely scenarios one could postulate the number of searchers and the number of targets in the search area as a function of time and then compute a probability of detection for each scenario. In this way the effects of various courses of action could be evaluated.

## APPENDIX A. DEVELOPMENT OF RANDOM SEARCH MODEL

This appendix contains the detailed derivation of the probability of detecting a target utilizing the random search model discussed in Chapter III. This derivation is based on the development in Naval Operations Analysis [Ref. 9:pp. 125-127]. Washburn [Ref. 8:pp. Chapter 2 1-4] derives the same formula using differential equations.

### A. RANDOM SEARCH MODEL

Assume that a target is randomly distributed in an area  $A$ , and that the search is conducted in a random manner. What is the probability that the target will be detected before the searcher has searched for  $T$  time periods or covered a total distance  $L$ ?

If the searcher travels at a velocity of  $V$ , he will cover a total distance  $L (= VT)$  by time  $T$ . Now divide the total search length  $L$  into  $N$  segments of equal length  $L/N$ . For detection to occur in the first segment, the target must be within the maximum sensor range  $R_m$  of the searcher, and the target must be detected. The probability that a target will be detected given that it is a certain range from the sensor is given by the sensor's lateral range curve. The lateral range curve can be replaced by a single quantity called the

sweep width,  $W$ . The sweep width "represents the effective width of the sensor's detection zone"[Ref. 9:p. 119] against the given target in the given environmental conditions. Therefore for a target to be detected it must be within an area of length  $L/N$  and width  $W$ . The probability of detecting the target in this area is found by dividing by the area by the total search area  $A$ .

$$P\{detection\} = \frac{WL}{NA}$$

The probability of not detecting the target in the first segment is  $1 - P\{detection\}$  or

$$P\{no\ detection\} = 1 - \frac{WL}{NA}$$

Now consider the entire search effort. The probability of not detecting the target over the entire search is the probability that the target is missed on all segments

$$\begin{aligned} P\{no\ detection_{search}\} &= \prod_{i=1}^N \left(1 - \frac{WL}{NA}\right) \\ &= \left(1 - \frac{WL}{NA}\right)^N \end{aligned}$$

Thus the probability of detecting the target over the entire search is



$$\begin{aligned}
 P\{detection_{search}\} &= 1 - P\{no\ detection_{search}\} \\
 &= 1 - \left(1 - \frac{WL}{NA}\right)^N
 \end{aligned}$$

This can be simplified by noting that for large N

$$\left(1 - \frac{\alpha}{N}\right)^N \approx e^{-\alpha}$$

Letting  $\alpha = WL/A$

$$\begin{aligned}
 P\{detection_{search}\} &= 1 - \left(1 - \frac{\alpha}{N}\right)^N \\
 &= 1 - e^{-\alpha}
 \end{aligned}$$

Then the probability of detecting a target in an area A during a search of total length L using a sensor with sweep width W is

$$P\{detection_{search}\} = 1 - e^{-WL/A} .$$

Similarly, the probability of detection in terms of the total search time T and searcher velocity V is

$$P\{detection_{search}\} = 1 - e^{-WVT/A} .$$

## APPENDIX B. PROBABILITY OF DETECTION BY CONDITIONING

From Chapter III the conditional probability of detecting an intermittently exposed target during a random search is given by

$$P\{\text{det}_{\text{search}}|\Phi = \phi\} = 1 - e^{-z\phi}$$

where  $z = WVT/A$  and  $W$  is the sweep width,  $V$  is the velocity of the searcher,  $T$  is the total time searching, and  $A$  is the search area.  $\Phi$  is the probability that the target will be exposed to detection when the searcher is within his maximum sensor range of the target.  $\Phi$  is a random variable, so the unconditional  $P\{\text{det}_{\text{search}}\}$  can be computed to conditioning on  $\Phi$  taking on specific values. To do this though,  $\Phi$  must be assumed to have a specific probability distribution function,  $f_{\Phi}(\phi)$ . We have assumed three possible distributions for  $\Phi$ , uniform  $(0,1)$ , triangular  $(0,1)$  with mean one-half, and triangular  $(0,1)$  with mean one-third.

### A. $\Phi$ DISTRIBUTED UNIFORMLY BETWEEN ZERO AND ONE

The unconditional probability is computed from the conditional probability and the distribution of  $\Phi$  [Ref. 10]. Let  $E$  be an arbitrary event like detecting a target and let  $Y$  be a continuous random variable with density  $f_Y(y)$ . Then

$$P(E) = \int_{-\infty}^{\infty} P(E|Y = y) f_Y(y) dy$$

or

$$P(\text{det}_{search}) = \int_{-\infty}^{\infty} P(\text{detection}|\Phi = \phi) f_{\Phi}(\phi) d\phi$$

For the Uniform (0,1) distribution

$$f_{\Phi}(\phi) = \begin{cases} 1 & 0 \leq \phi \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Therefore the unconditional probability of detection can be computed as

$$\begin{aligned} P(\text{det}_{search}) &= \int_0^1 (1 - e^{-z\phi}) d\phi \\ &= 1 - \frac{1}{z} + \frac{e^{-z}}{z} \end{aligned}$$

## B. $\Phi$ DISTRIBUTED TRIANGULARLY WITH MEAN ONE-HALF

Now assume  $\Phi$  has a triangular distribution between zero and one, and a mean of one-half. Figure 11 shows the probability density function of the triangular distribution. The density function is

$$f_{\Phi}(\phi) = \begin{cases} 4\phi & 0 \leq \phi \leq \frac{1}{2} \\ 4 - 4\phi & \frac{1}{2} \leq \phi \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Therefore the unconditional probability of detection is

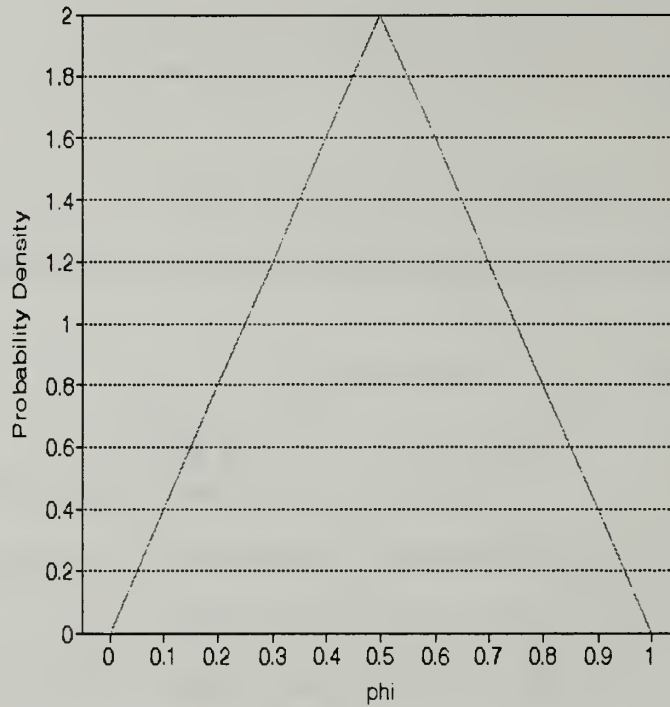


Figure 11. pdf of Triangular Distribution Mean One-Half.

$$\begin{aligned}
 P\{\det_{search}\} &= \int_0^{\frac{1}{2}} (1 - e^{-z\phi}) (4\phi) d\phi + \\
 &\quad \int_{\frac{1}{2}}^1 (1 - e^{-z\phi}) (4 - 4\phi) d\phi \\
 &= \frac{1}{z^2} (z^2 + 8e^{-\frac{z}{2}} - 4 - 4e^{-z})
 \end{aligned}$$

### C. $\Phi$ DISTRIBUTED TRIANGULARLY WITH MEAN ONE-THIRD

Finally, assume that  $\Phi$  is again distributed triangularly between zero and one but this time the mean is one-third. Figure 12 shows the probability density function of the distribution. Mathematically, the density function is

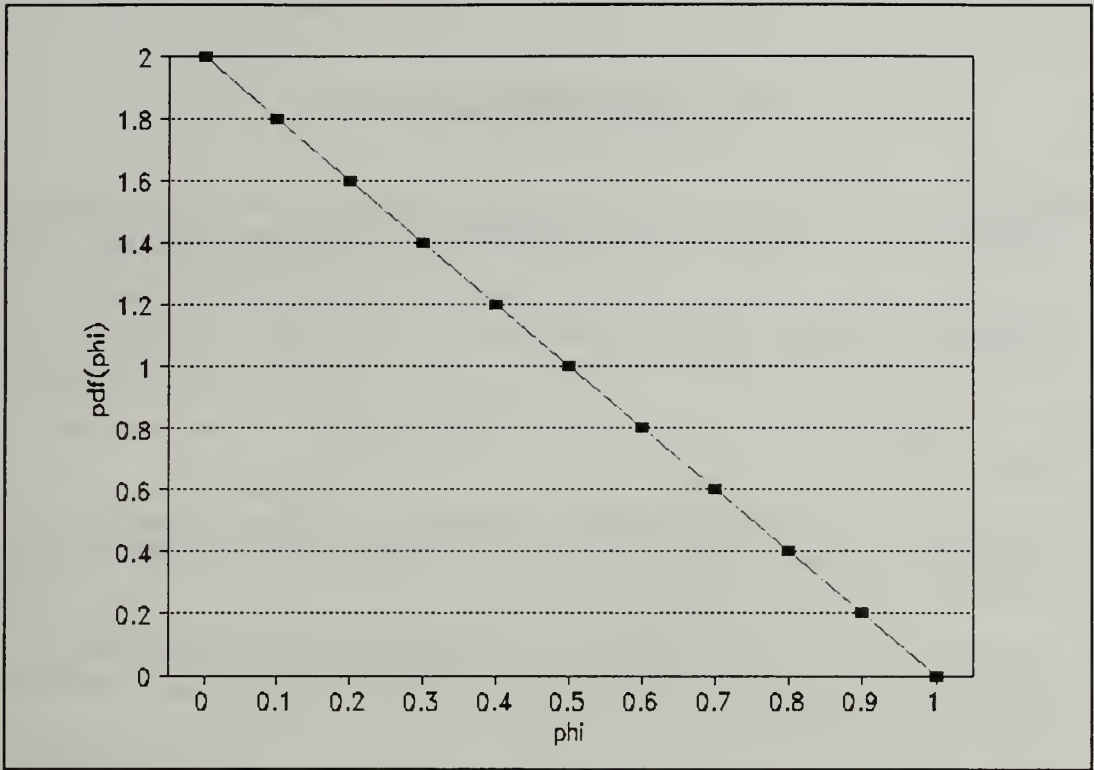


Figure 12. pdf of Triangle Distribution Mean One-third.

$$f_{\Phi}(\phi) = \begin{cases} 2 - 2\phi & 0 \leq \phi \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The unconditional probability of detection when  $\Phi$  has a triangular distribution with a mean of one-third is

$$\begin{aligned} P\{\text{det}_{search}\} &= \int_0^1 (1 - e^{-z\phi})(2 - 2\phi) d\phi \\ &= \frac{1}{z^2} (z^2 - 2e^{-z} - 2z + 2) \end{aligned}$$

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